

Lunar Cratering - Probability and Odds



The moon has lots of craters! If you look carefully at them, you will discover that many overlap each other. Suppose that over a period of 100,000 years, four asteroids struck the lunar surface. What would be the probability that they would strike an already-cratered area, or the lunar mare, where there are few craters?

Problem 1 - Suppose you had a coin where one face was labeled 'C' for cratered and the other labeled U for uncratered. What are all of the possibilities for flipping C and U with four coin flips?

Problem 2 - How many ways can you flip the coin and get only Us?

Problem 3 - How many ways can you flip the coin and get only Cs?

Problem 4 - How many ways can you flip the coin and get 2 Cs and 2 Us?

Problem 5 - Out of all the possible outcomes, what fraction includes only one 'U' as a possibility?

Problem 6 - If the fraction of desired outcomes is $\frac{2}{16}$, which reduces to $\frac{1}{8}$, we say that the 'odds' for that outcome are 1 chance in 8. What are the odds for the outcome in Problem 4?

A fair coin is defined as a coin whose two sides have equal probability of occurring so that the probability for 'heads' = $\frac{1}{2}$ and the probability for tails = $\frac{1}{2}$ as well. This means that $P(\text{heads}) + P(\text{tails}) = \frac{1}{2} + \frac{1}{2} = 1$. Suppose a tampered coin had $P(\text{heads}) = \frac{2}{3}$ and $P(\text{tails}) = \frac{1}{3}$. We would still have $P(\text{heads}) + P(\text{tails}) = 1$, but the probability of the outcomes would be different...and in the cheater's favor. For example, in two coin flips, the outcomes would be HH, HT, TH and TT but the probabilities for each of these would be $HH = (\frac{2}{3}) \times (\frac{2}{3}) = \frac{4}{9}$; HT and TH = $2 \times (\frac{2}{3})(\frac{1}{3}) = \frac{4}{9}$, and $TT = (\frac{1}{3}) \times (\frac{1}{3}) = \frac{1}{9}$. The probability of getting more heads would be $\frac{4}{9} + \frac{4}{9} = \frac{8}{9}$ which is much higher than for a fair coin.

Problem 7: From your answers to Problem 2, what would be the probability of getting only Us in 4 coin tosses if A) $P(U) = \frac{1}{2}$? B) $P(U) = \frac{1}{3}$?

Problem 8 - The fraction of the lunar surface that is cratered is $\frac{3}{4}$, while the mare (dark areas) have few craters and occupy $\frac{1}{4}$ of the surface area. If four asteroids were to strike the moon in 100,000 years, A) what is the probability that all four would strike the cratered areas?

Answer Key

4

Problem 1 - The 16 possibilities are as follows:

C U U U	C C U U	U C U C	C U C C
U C U U	C U C U	U U C C	U C C C
U U C U	C U U C	C C C U	C C C C
U U U C	U C C U	C C U C	U U U U

Note if there are two outcomes for each coin flip, there are $2 \times 2 \times 2 \times 2 = 16$ independent possibilities.

Problem 2 - There is only one outcome that has 'U U U U'

Problem 3 - There is only one outcome that has 'C C C C'

Problem 4 - From the tabulation, there are 6 ways to get this outcome in any order.

Problem 5 - There are 4 outcomes that have only one U out of the 16 possible outcomes, so the fraction is $4/16$ or $1/4$.

Problem 6 - The fraction is $6 / 16$ reduces to $3/4$ so the odds are 3 chances in 4.

Problem 7: A) If each U has a probability of $1/2$, then the probability is $1/16 \times (1/2) \times (1/2) \times (1/2) \times (1/2) = 1/(16 \times 16) = 1/256$.

B) If each U has a probability of $1/3$, then the probability is $1/16 \times (1/3) \times (1/3) \times (1/3) \times (1/3) = 1/(16 \times 81) = 1/1296$.

Problem 8 - $P(U) = 1/4$ while $P(C) = 3/4$, so the probability that all of the impacts are in the uncratered regions is the outcome C C C C which is $1/16$ of all possible outcomes, so its probability is $1/16 \times (3/4) \times (3/4) \times (3/4) \times (3/4) = 81 / 4096 = 0.0198$.